A new MDP model for self-similar call traffic with application to call admission control

Ernst Nordström

Department of Culture/Media/Computer Science, Dalarna University

SE-781 88 Borlänge, Sweden

Email: eno@du.se

Abstract

In this paper we study the call admission control (CAC) problem for a single link in multi-service loss networks. Each call is described by a reward parameter representing the expected reward for carrying this call. The control objective is to maximize the reward from carried calls.The behavior of the link is modeled as a Markov Decision Process (MDP). The standard link MDP model assumes a Poisson call arrival process and exponentially distributed call holding times. However, some services on the Internet, such as the World Wide Web service, produce self-similar call arrival processes with heavy-tailed holding time distributions. In this paper, we propose an extended link MDP model for self-similar call arrivals and exponential holding time distributions. Numerical results show that the reward increase due to admission of a call (denoted link net-gain) depends on the time offset between the latest arrival and the latest departure.

1 INTRODUCTION

We consider the problem of Call Admission Control (CAC) on a single link in multi-service loss networks such as ATM and STM networks, and IP networks, provided they are extended with resource reservation capabilities. The objective is to maximize the revenue from carried calls, while meeting constraints on the Quality of Service (QoS) and Grade of Service (GoS) on the packet and call level, respectively. First, the network should determine the set of feasible paths between the source and destination which offers sufficient QoS to the new and existing calls in terms of delay, jitter and data loss. Second, the network should chose to reject the call or to accept it on some path among the set of feasible paths. This choice should be consistent with GoS constraints, in terms of call blocking probabilities and call set up delays, as well as maximizing the average revenue rate for the operator.

The required bandwidth is represented by the call's peak bandwidth in case of deterministic multiplexing, and by the call's equivalent bandwidth in case of statistical multiplexing. Note that the equivalent bandwidth can be different on different links along the call's path. In particular, the equivalent bandwidth depends on the traffic mix on the link, buffer and link capacity as well as the target buffer overflow probability.

Modern CAC and routing mechanisms are state-dependent rather than static, which means that the decision to reject the request for a new call, or to accept it on a particular path depends on the current occupancy of the network. A state-dependent CAC and routing policy is a mapping, for every call class, from a network state space to a set of possible CAC and routing decisions, see Figure 1. State-dependent mechanisms offer advantages both in terms of achievable revenue and ability to control the QoS and GoS.

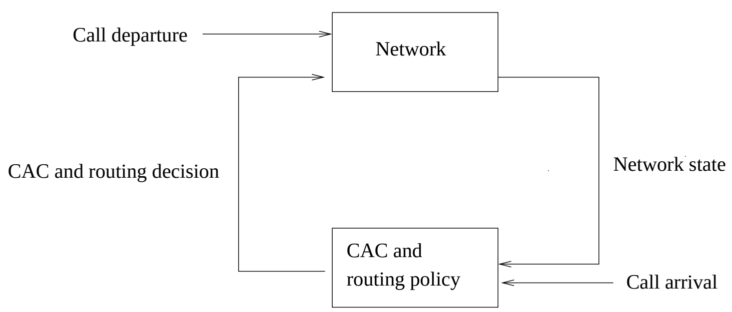

Figure 1: State-dependent CAC and routing

This paper deals with a particular form of state-dependent CAC on the link level, where the behavior of the link is formulated as Markov Decision Process (MDP) [3, 16] . A MDP is a controlled Markov process, where the set of state transitions from the current Markov state to other Markov states depends on the decision or action taken by the controller in the current state. In the MDP framework, each call is described by a expected reward parameter and the objective is to maximize the reward from carried calls.

Optimal state-dependent CAC and routing policies can be computed using an exact network MDP framework. However, the cardinality of the network state and policy spaces in the exact framework can be very large even for moderate-size networks. Therefore, a necessary modelling simplification is to decompose the network into a set of links assumed to have independent traffic and reward processes, respectively. When formulating the MDP framework for each link, calls with the same bandwidth requirement are aggregated into a common category, which corresponds to one dimension in the link state vector.

The standard link MDP model assumes Poisson call arrival processes and exponentially distributed call holding times. The Poisson call arrival model is accurate for session arrivals for many service types, such as telephony, World Wide Web (WWW), FTP and TELNET. However, for TCP connections invoked within the WWW sessions, the Poisson model is an inadequate model. Based on real measurements of the TCP connection arrival process within WWW sessions, Anja Feldmann has proposed a certain non-Poisson renewal call arrival process model [6, 7, 8]. This process has inter-arrival times that follow a Weibull distribution in contrast to the Poisson process which has exponentially distributed inter-arrival times. We follow the convention by Anja Feldmann and refer to this new arrival process as self-similar, since it exhibits structural similarities across a wide range of time scales. The Weibull distribution is a generalized exponential distribution. For the range of distribution parameters plausible for TCP connection arrivals within WWW sessions, the complementary Weibull distribution decays slower (has a more heavy tail) than the standard exponential distribution. Measurements have also shown that the complementary distribution of holding times of TCP connections within WWW sessions decays slower than exponentially [2].

In case of a Poisson arrival process, and exponential service process, the state transition probabilities, which are part of the MDP model, become easy to formulate. This is due to the memoryless property of the exponential distribution: the probability of the next event being an arrival/departure is independent of the time offset between the latest arrival and the latest departure. This is not the case if we replace the Poisson process with a non-Poisson process such as the above self-similar process: the probability of the next event being an arrival/departure now becomes dependent on the time between the latest arrival and the latest departure.

Generally, when a Poisson arrival process is multiplexed with a Markov (exponential) service process, the resulting superposed process is a renewal process. In this case, the controlled sequence of link states forms a semi-Markov decision process (SMDP) [16]. However, when a renewal arrival process is multiplexed with a Markov (exponential) service process, the resulting superposed process is not renewal. In order to formulate a SMDP it is necessary to introduce new state variables, which contains enough information for accurate prediction of future state vectors (resulting in preservation of the Markov property).

In a previous paper [11] we analyzed the performance of the standard link MDP model applied for both Poisson and self-similar call arrivals. The simulation results showed that the direct application of the standard link MDP model to the unexpected case with self-similar call arrivals, yielded fairly good performance in terms of average reward loss.

This paper proposes an extended link MDP model for self-similar call arrivals and exponential holding time distributions. We limit ourselves to exponential holding time distributions since this case is easier to handle than the case with heavy-tailed holding time distributions. Moreover, we consider only the one call category case. However, the model can easily be generalized to cope with multiple categories, albeit with large computational complexity. Numerical results show that the reward increase due to admission of a call (denoted link net-gain) depends on the time offset between the latest arrival and the latest departure. We expect that the performance of CAC and routing on the network level can be improved by extending the standard link MDP model.

The paper is organized as follows. Section 2 formulates the CAC problem in terms of offered traffic and optimization objective. Section 3 describes different models of the call arrival process and call holding time distribution. Section 4 describes the link MDP model for self-similar call arrivals. Section 5 outlines the MDP computation procedure for self-similar call arrivals. Section 6 evaluates the standard and extended link MDP model using numerical/simulation techniques. Section7 discuss how to deal with multiple call categories, as well as CAC and routing on the network level. Finally, Section 8 concludes the paper.

2 PROBLEM FORMULATION

We consider a single communication link with capacity C Mbps. The link is offered traffic from G categories which are, for sake of simplicity, assumed to be subject to deterministic multiplexing. The i-th category, $i \in I = \{1, \cdots, G\}$, is characterized by the following:

- Peak bandwidth requirement b_i [Mbps],
- General call arrival process $\{A_k\}$ with two special cases:
  - Poisson process with average arrival rate λ_i [s^{-1}],
  - Self-similar process characterized by Weibull parameters a_i and c_i,
- Exponential service process $\{B_k\}$ with mean 1/μ_i [s],
- Reward parameter $r_i \in (0, \infty)$

The task is to find an optimal link CAC policy π^* which maximizes the mean reward from the link, defined as

$$\overline{R}(\pi) = \sum_{i \in I} r_i \overline{\lambda}_i \tag{1}$$

where $\overline{\lambda}_i$ denotes the average category-i acceptance rate.

3 MODELLING OF CALL TRAFFIC

3.1 Call arrival process

Since the days of Erlang the Poisson model has commonly been used to describe the random arrivals of call requests to the OD pairs of a telephone network. Although the Poisson model serves its purpose in telephone networks, it lacks descriptive power in the case of Internet where a substantial portion of traffic is World Wide Web (WWW) connections transported by TCP. The nature of the WWW service is different from the telephone service; A person using the WWW service is more likely to initiate additional downloads after the first download. A person using the telephone service is more likely to initiate independent calls.

Measurements on real WWW connection arrivals in the Internet has revealed that the arrival process shows burstiness over many time scales, ranging from seconds to hours. Anja Feldmann [7, 8] argues that this traffic is self-similar. The degree of burstiness over different time scales or the extend of self-similarity can be expressed with just one single parameter, the Hurst parameter. For self-similar processes its value is between 0.5 and 1 and the degree of self-similarity increases as the Hurst parameter approaches 1. Together with the Poisson nature of WWW session arrivals, the empirically observed property that the number of TCP connections per WWW session is heavy tailed with indications of infinite variance provides a mathematical explanation for the self-similar nature of WAN traffic at the TCP level.

More formal, a covariance-stationary process $X = \{X_k : k \geq 1\}$ is called *asymptotically self-similar* (with self-similarity parameter H, $0 < H < 1$), for a large enough m,

$$X_k \stackrel{d}{=} m^{1-H} X_k^{(m)}, \tag{2}$$

where $X^{(m)} = (X_k^{(m)} : k \geq 1)$ is the *aggregated* process of order (time scale) m, given by

$$X_k^{(m)} = \frac{1}{m}(X_{(k-1)m+1} + \cdots + X_{km}), k \geq 1. \tag{3}$$

The process under consideration is the number of call arrivals per time unit.

Anja Feldmann has observed that the WWW connection arrivals can be accurately modeled by a renewal process with inter-arrival times that follow a Weibull distribution, $A(t) = P(A_k \leq t) = 1 - e^{-\left(\frac{t}{a}\right)^c}$. Recall that the Poisson process has exponentially distributed inter-arrival times: $A(t) = P(A_k \leq t) = 1 - e^{-\lambda t}$. Hence, the Weibull distribution can be seen as a generalized exponential distribution. When $c = 1$ the Weibull distribution becomes identical to the standard exponential distribution.

The exponential probability density function (pdf) is

$$a(t) = \frac{dA(t)}{dt} = \frac{d}{dt}(1 - e^{-\lambda t}) = \lambda e^{-\lambda t}. \tag{4}$$

The mean inter-arrival time for the Poisson process is:

$$E[A_k] = \int_0^\infty t a(t) dt = \int_0^\infty t \lambda e^{-\lambda t} dt = \frac{1}{\lambda}. \tag{5}$$

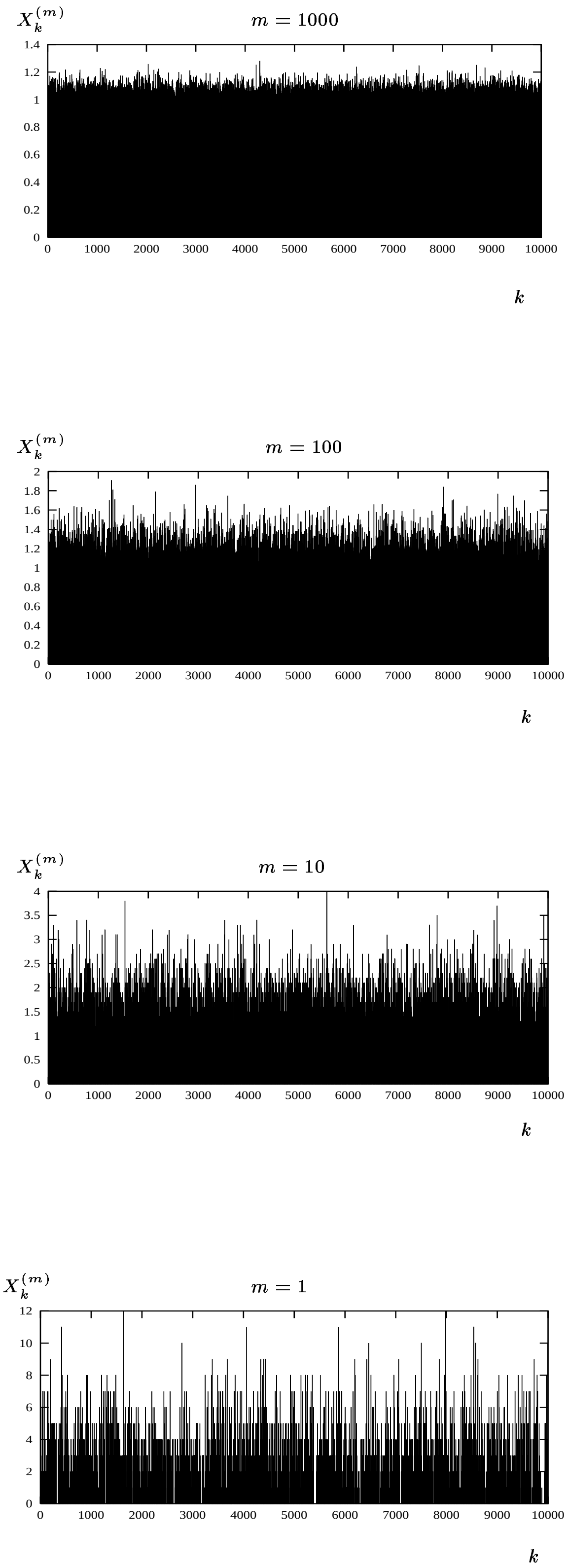

Figure 2: Self-similar call arrivals

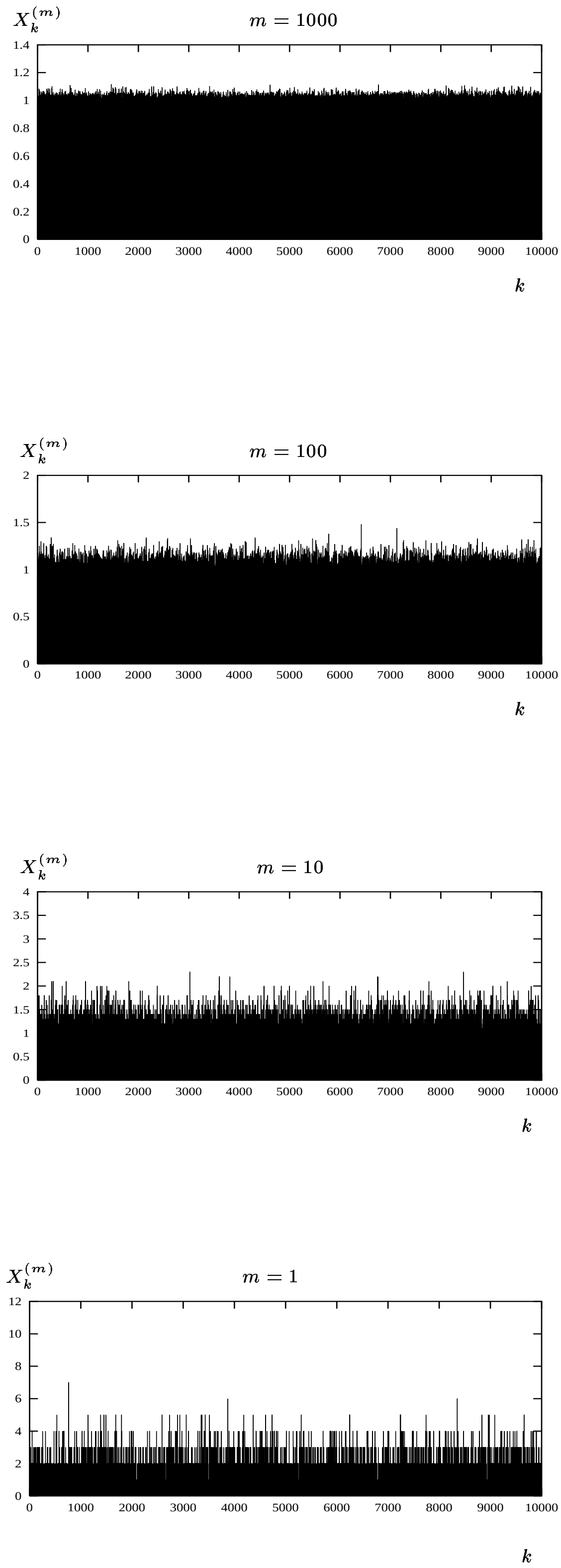

Figure 3: Poisson call arrivals

The Weibull pdf is

$$a(t) = \frac{dA(t)}{dt} = \frac{d}{dt}(1 - e^{-\left(\frac{t}{a}\right)^c}) = \frac{c}{a}\left(\frac{t}{a}\right)^{c-1} e^{-\left(\frac{t}{a}\right)^c}. \tag{6}$$

The mean inter-arrival time for the Weibull-based process is:

$$E[A_k] = \int_0^\infty ta(t)dt = \int_0^\infty t\frac{c}{a}\left(\frac{t}{a}\right)^{c-1} e^{-\left(\frac{t}{a}\right)^c} dt \tag{7}$$

$$\left\{ \begin{array}{l} s = \left(\frac{t}{a}\right)^c, t = as^{1/c} \\ ds = c\left(\frac{t}{a}\right)^{c-1} dt \\ dt = \frac{a}{c}s^{\frac{1-c}{c}} ds \end{array} \right\} = c\int_0^\infty se^{-s}\frac{a}{c}s^{\frac{1-c}{c}} ds \tag{8}$$

$$= a\int_0^\infty s^{1/c}e^{-s}ds = a\Gamma(1 + \frac{1}{c}). \tag{9}$$

Figure 2 and Figure 3 shows the aggregate arrival process $X_k^{(m)}$ for different time scales m for the self-similar process and the Poisson process. In the Figures, the mean arrival rate λ of the Poisson process where chosen to be 1 [s^{-1}], and the Weibull parameters were $c = 0.5$ and $a = \frac{1}{\Gamma(1+\frac{1}{c})\lambda}$. Obviously, the variability of the aggregate process decreases faster for the Poisson process than the self-similar process when the time scale is increasing.

For a self-similar process the variance of the aggregated process decays like

$$Var[X_k^{(m)}] \backsim m^{-\beta}, \tag{10}$$

where $\beta = 2(1 - H)$. The variance-time plot is a popular method for determining β and thus the Hurst parameter $H = 1 - \beta/2$. One simply plots $\log_{10} Var[X_k^{(m)}]$ against $\log_{10}(m)$ and then determines the slope $-\beta$. Figure 4 shows the variance-time plot for the studied Poisson process and the self-similar process. Note that the logarithm of the variance drops with a slope of -1 for Poisson process and with a slope $-\beta, 0 < \beta < 1$, for the self-similar process.

Finally, we show in Figure 5, for the studied self-similar process, the Hurst parameter H as function of the Weibull parameter c. Note that the Hurst parameter does not depend on the Weibull parameter a.

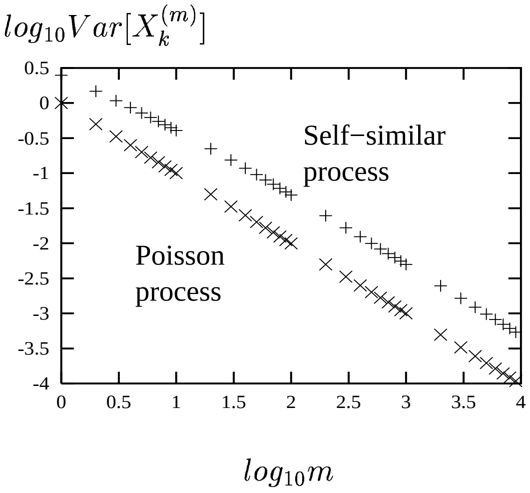

Figure 4: Variance-time plot

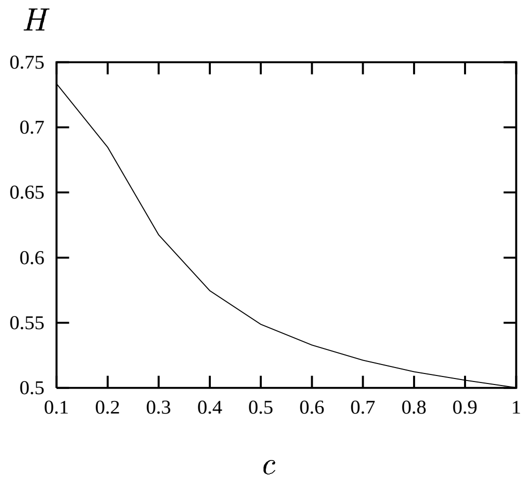

Figure 5: Hurst parameter vs. c

3.2 Call holding time distribution

The traditional model of call holding times B_k is the (negative) exponential distribution with rate parameter μ: $B(t) = \mathrm{P}(B_k \leq t) = 1 - \exp(-\mu t)$. The exponential distribution match the actual holding times in case of telephony among other services. The associated holding time pdf is

$$b(t) = \frac{dB(t)}{dt} = \frac{d}{dt}(1 - e^{-\mu t}) = \mu e^{-\mu t}. \tag{11}$$

The distribution function for the inter-departure times for a group of n calls is given by $B_n(t) = \mathrm{P}(B_k^n \leq t) = 1 - \exp(-n\mu t)$. Similarly, the pdf for the inter-departure times for the group of n calls is:

$$b_n(t) = \frac{dB_n(t)}{dt} = \frac{d}{dt}(1 - e^{-n\mu t}) = n\mu e^{-\mu t}. \tag{12}$$

However, for TCP connections invoked within WWW sessions the holding time is more heavy tailed [2]. The reason is that the distribution of WWW document sizes on the Internet is heavy tailed. A distribution is said to be heavy tailed if Prob$[X > x] \sim cx^{\alpha}$ with $\alpha > 0$. Intuitively, a heavy-tailed holding time distribution means that if the call has not been completed for some time it becomes more and more unlikely that it will be completed soon. The Pareto distribution is a popular choice for the heavy tailed distribution. It has the following form: $B(t) = \mathrm{P}(B_k \leq t) = 1 - \left(\frac{a}{t}\right)^{\alpha}, a > 0$.

4 LINK MODEL FOR SELF-SIMILAR CALL ARRIVALS: ONE CATEGORY CASE

In this section we formulate the link MDP model for the one category ($G = 1$) case, assuming a general renewal call arrival process and an exponential holding time distribution. Let $\mathbf{x} = (n, e, \omega)$ denote the current state of the MDP, and let $\mathbf{y} = (m, f, \Omega)$ denote the MDP state which is entered after an event in the current state. The variables n and m denote link state representing the number of active calls from the single category. The variables e and f denote the event type (*ARRIVAL/DEPARTURE*) of the latest event. The variables ω and Ω denote the (probability mass) offset between an event of type e and f, respectively, and the latest complementary event. The complement of an *ARRIVAL* event is a *DEPARTURE* event and vice versa.

The state space X is given by:

$$X = \{\mathbf{x} = (n, e, \omega) : n = 0, 1 \cdots, \lfloor C/b \rfloor; e \in \{\textit{ARRIVAL,DEPARTURE}\}; \\ \omega = 0, 1 \cdots, N_\omega - 1\}, \qquad (13)$$

where C denotes the link capacity and b denotes the call's bandwidth requirement.

The action space is given by

$$A = \{\theta \in \{0, 1\}\}, \qquad (14)$$

where $\theta = 0$ denotes call rejection and $\theta = 1$ denotes call acceptance. The permissible action space is a state-dependent subset of A:

$$A(\mathbf{x}) = \{\theta \in A : \theta = 0 \text{ if } n + 1 > \lfloor C/b \rfloor\}. \qquad (15)$$

The state transition probabilities are given by:

$$p_{xy}(\theta) = \begin{cases} \int_{t_u+\zeta_e}^{t_u+\zeta_e+\Delta_u} \theta a(t)\phi(t)dt, & m = n+1, \Omega = \Omega_{ef}(u, \omega), u \in \{0, 1 \cdots, N_\omega - 1\} \\ \int_{t_u+\zeta_e}^{t_u+\zeta_e+\Delta_u} (1-\theta)a(t)\phi(t)dt, & m = n, \Omega = \Omega_{ef}(u, \omega), u \in \{0, 1 \cdots, N_\omega - 1\} \\ \int_{t_u+\xi_e}^{t_u+\xi_e+\Delta_u} b_n(t)\varphi(t)dt, & m = n-1, \Omega = \Omega_{ef}(u, \omega), u \in \{0, 1 \cdots, N_\omega - 1\} \\ 0 & \text{otherwise,} \end{cases} \qquad (16)$$

where

$$\phi(t) = [1 - B_n(t + \xi_e)]/[1 - A(\zeta_e)], \tag{17}$$

$$\varphi(t) = [1 - A(t + \zeta_e)]/[1 - B_n(\xi_e)], \tag{18}$$

where

$$\xi_e = \begin{cases} \Delta_\omega(e), & e = \textit{ARRIVAL} \\ 0, & e = \textit{DEPARTURE}, \end{cases} \tag{19}$$

$$\zeta_e = \begin{cases} \Delta_\omega(e), & e = \textit{DEPARTURE} \\ 0, & e = \textit{ARRIVAL}, \end{cases} \tag{20}$$

and

$$\Delta_\omega(e) = \begin{cases} \frac{1}{n\mu}\left[-\log(1 - \omega/N_\omega)\right], & e = \textit{ARRIVAL} \\ a\left[-\log(1 - \omega/N_\omega)\right]^{1/c}, & e = \textit{DEPARTURE}. \end{cases} \tag{21}$$

Note that the time offset $\Delta_\omega(e)$ is an increasing function of the probability mass offset ω. Moreover, in the case of exponentially distributed inter-arrival times (Poisson) and exponentially distributed service times, the memoryless property of the exponential distribution allow us to put $\xi_e = \zeta_e = 0$.

The offset Ω in the new state $\mathbf{y}$ is given by

$$\Omega_{ef}(u, \omega) = \begin{cases} \lfloor B_n(t_u + \Delta_u)N_\omega \rfloor, & e = \textit{ARRIVAL}, f = \textit{DEPARTURE} \\ \lfloor A(t_u + \Delta_u)N_\omega \rfloor, & e = \textit{DEPARTURE}, f = \textit{ARRIVAL} \\ \lfloor B_n(t_u + \Delta_u + \Delta_\omega(e))N_\omega \rfloor, & e = f = \textit{ARRIVAL} \\ \lfloor A(t_u + \Delta_u + \Delta_\omega(e))N_\omega \rfloor, & e = f = \textit{DEPARTURE}. \end{cases}$$

The expected sojourn time $\tau(\mathbf{x})$ in state $\mathbf{x}$ is determined as

$$\tau(\mathbf{x}) = \int_{\zeta_e}^{\infty} (t - \zeta_e)a(t)\phi(t)dt + \int_{\xi_e}^{\infty} (t - \xi_e)b_n(t)\varphi(t)dt. \tag{22}$$

The expected immediate reward $R(\mathbf{x})$ in state $\mathbf{x}$ is given by:

$$R(\mathbf{x}) = r \int_{\xi_e}^{\infty} b_n(t)\varphi(t)dt, \tag{23}$$

where r denotes the reward parameter for the single category.

We now describe one way to determine the Δ_u values used in the expression for the state transition probabilities. For modelling convenience we replace the self-similar arrival process with a Poisson process. The quality of the final solution is not critically dependent on the Δ_u values; some deviation from the Δ_u values based on the self-similar arrival process is tolerated.

The probability of a call arrival within an infinite interval is:

$$P(\text{arrival in } (0,\infty)) = \int_0^{\infty} \lambda e^{-[\lambda+n\mu]t}dt = \lambda\tau(n), \tag{24}$$

where $\tau(n)$ is the mean sojourn time in link state n:

$$\tau(n) = [\lambda + n\mu]^{-1}. \tag{25}$$

The probability for an arrival (assuming the offset is zero) during $[t_u, t_u + \Delta_u]$ can be written:

$$P(\text{arrival in } (t_u, t_u+\Delta_u)) = \int_{t_u}^{t_u+\Delta_u} \lambda e^{-[\lambda+n\mu]t}dt = \lambda\tau(n)e^{-t_u/\tau(n)}[1 - e^{-\Delta_u/\tau(n)}] \tag{26}$$

Let us choose Δ_u such that this probability becomes $1/N_\omega$ times the probability of arrival within an infinite interval. This gives:

$$\Delta_u = -\tau(n)\log(1 - \frac{1}{N_\omega}e^{t_u/\tau(n)}). \tag{27}$$

In order to determine the state transition probabilities, expected sojourn times, and expected immediate rewards, we use numerical integration, e.g. the Simpson's method.

5 MDP COMPUTATIONAL PROCEDURE FOR SELF-SIMILAR CALL ARRIVALS

This section outlines the MDP computational procedure for determining an optimal CAC policy using the link model described in Section 4. The *link net-gain function* plays an important role in MDP-based CAC and routing. For the self-similar link model, when we have an arrival, the state jumps from the current state (n, e, ω) to the request state $(n, \mathit{ARRIVAL}, \Omega)$. If the CAC controller decides to accept the request the new state will become $(n+1, \mathit{ARRIVAL}, \Omega)$, otherwise the state $(n, \mathit{ARRIVAL}, \Omega)$ is maintained. The link net-gain function is defined as the increase in long-term reward due to admission of a call in link state $(n, \mathit{ARRIVAL}, \Omega)$:

$$g(n, \Omega, \pi) = v(n+1, \mathit{ARRIVAL}, \Omega, \pi) - v(n, \mathit{ARRIVAL}, \Omega, \pi), \tag{28}$$

where $v(n, \mathit{ARRIVAL}, \Omega, \pi)$ denotes the *relative value* for the single category in state $(n, \mathit{ARRIVAL}, \Omega)$. Note that the link CAC controller accepts the call request as long as the link net-gain is positive, and it rejects the call request when the link net-gain is negative.

To give more insight into the definition of relative values, let us define the expected link reward, $R(x_0, \pi, T)$, obtained in a interval $(t_0, t_0 + T)$ of length T, assuming state x_0 at time t_0:

$$R(x_0, \pi, T) = E\left[\int_{t_0}^{t_0+T} q(\mathbf{x}(t))dt\right], \tag{29}$$

where $q(\mathbf{x}(t))$ denotes the reward accumulation rate in state $\mathbf{x}(t)$. The process $\{\mathbf{x}(t)\}$ is driven by a probabilistic law of motion specified by certain state transition probabilities.

The relative value can now be written as:

$$v(\mathbf{x}_0, \pi) = \lim_{T\to\infty} \left[R(\mathbf{x}_0, \pi, T) - R(\mathbf{x}_r, \pi, T)\right]. \tag{30}$$

That is, the relative value in state $\mathbf{x}_0$ is defined as the difference in future reward earnings when starting in the given state, compared to a reference state, $\mathbf{x}_r$. In practice, the relative value function is obtained by solving a set of linear equations (see below).

In the context of the link MDP model outlined in the previous section, the algorithm for determining the optimal CAC policy π and the associated relative values $v(\mathbf{x}, \pi)$ and average reward rate $\overline{R}(\pi)$ can be summarized as follows:

1. **Initialization**: Choose an initial CAC policy π, e.g. the complete sharing policy

2. **Value determination**: Find the relative values $v(\mathbf{x}, \pi)$ and average reward rate $\overline{R}(\pi)$ for the current CAC policy π

3. **Policy improvement**: Improve the link CAC policy π based on the new relative values and the new average reward rate

4. **Convergence test**: Repeat from 2 until average reward per time unit converges.

According to MDP theory an optimal policy is found after a finite number of policy iterations in case of a finite state and policy space [16].

The *value determination step* determines the average reward rate $\overline{R}(\pi)$ and relative values $v(\mathbf{x}, \pi)$ for all states $\mathbf{x} \in X$ by solving a sparse system of linear equations:

$$\begin{cases} v(\mathbf{x}, \pi) = R(\mathbf{x}) - \overline{R}(\pi)\tau(\mathbf{x}) + \sum_{\mathbf{y} \in X} p_{xy}(\theta) v(\mathbf{y}, \pi) \\ v(\mathbf{x}_r, \pi) = 0; \qquad \mathbf{x} \in X \setminus \{\mathbf{x}_r\}, \end{cases} \tag{31}$$

where the following quantities need to be specified:

- X: the state space, i.e. the set of possible states,
- $\theta = \pi(\mathbf{x})$: the control action in state $\mathbf{x}$,
- $\tau(\mathbf{x})$: the expected sojourn time in state $\mathbf{x}$,
- $R(\mathbf{x})$: the expected link reward when leaving state $\mathbf{x}$,
- $p_{xy}(\theta)$: the transition probability from state $\mathbf{x}$ to state state $\mathbf{y}$, given that action θ is taken in state state $\mathbf{x}$,
- $\mathbf{x}_r$: the reference state (e.g. the empty state),

in order to compute the unknowns:

- $v(\mathbf{x}, \pi)$: the relative value in state $\mathbf{x}$ under CAC policy π,
- $\overline{R}(\pi)$: the average rate of link reward under policy π.

The computation (time) complexity of the value determination step of policy iteration is a function of the size, S, of the state space. Traditional Gauss elimination has complexity $\mathrm{O}(S^3)$. This can be seen as an upper limit of the actual complexity since the system is sparse and more efficient iterative algorithms can be used.

The *policy improvement step* consists of finding the action that maximizes the relative value in each state $\mathbf{x} \in X$:

$$\theta = \underset{\theta \in A(\mathbf{x})}{\mathsf{argmax}} \left\{ R(\mathbf{x}) - \overline{R}(\pi)\tau(\mathbf{x}) + \sum_{y \in X} p_{xy}(\theta) v(\mathbf{y}, \pi) \right\}, \tag{32}$$

where $A(\mathbf{x})$ denotes the set of possible actions in state $\mathbf{x}$. The set of actions which yields the maximum improvement of relative values constitute an improved policy π' to be used again in the value determination step. The policy improvement step has complexity $\mathrm{O}(2^G S)$, where G denotes the number of unique bandwidth categories.

6 NUMERICAL RESULTS

6.1 Considered link models

The performance analysis is performed for the single link case. Two MDP models for CAC are compared numerically:

- MDP – standard link model assuming Poisson call arrivals [5],
- MDP+ – extended link model assuming self-similar call arrivals according to Section 4.

6.2 Examples and results

The simulation scenario is described in Table 1. The traffic parameters are chosen such that the link load becomes moderate (83 % of link capacity). Each measurement period is based on 2×10^6 call events.

Two types of call arrival models are used the the simulations: the Poisson model (po) and the self-similar model (ss). The mean Poisson arrival rate λ and the mean holding time $1/\mu$ are first chosen. The self-similar call arrival process is chosen as follows. First, the Weibull

link capacity C [Mbps]	24
#traffic categories G	1
call arrival rate λ [s^{-1}]	20
mean holding time $1/\mu$ [s]	1
bandwidth b [Mbps]	1
link traffic [Mbps*Erlang]	20
reward parameter r	1
#offset values N_ω	10

Table 1: Description of simulation scenario

parameter c is set to $c = 0.5$ which is plausible value for real WWW connection arrivals [7, 8]. The choice $c = 0.5$ gives a Hurst parameter of $H = 0.55$, see Figure 5. Second, the Weibull parameter a is set to $a = \frac{1}{\Gamma(1+\frac{1}{c})\lambda}$ which gives equal mean inter-arrival time for the Poisson and self-similar process.

Table 2 shows the reward loss as an average over $M = 20$ simulation runs. The reward loss L in each simulation run is computed as

$$L = 1 - \overline{R}/R, \tag{33}$$

where $\overline{R} = \sum_{i \in I} r_i \overline{\lambda}_i$ and $R = \sum_{i \in I} r_i \lambda_i$ denotes the carried and offered reward rate, respectively.

The average reward loss is computed as $\overline{L} = \frac{1}{M} \sum_{i=1}^{M} L_i$, where L_i denotes the reward loss for simulation i. For assessment of the accuracy of the simulation results we present values of the standard deviation of the reward loss in the same table. We compute the standard deviation as:

$$s = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (L_i - \overline{L})^2}. \tag{34}$$

Figures 6 and 7 show the relative value function and the link net-gain function, respectively, for the standard MDP model assuming Poisson and self-similar call arrival processes.

Figures 8 – 11 show the relative values in different states (n, e, ω) for the extended link model (MDP+) assuming Poisson call arrival process.

	Reward loss in % (sdev)
MDP po	6.61 (0.06)
MDP ss	15.37 (0.10)
MDP+ po	6.61 (0.06)
MDP+ ss	15.32 (0.13)

Table 2: Reward loss for the standard (MDP) and the extended (MDP+) methods for Poisson and self-similar call arrivals, respectively.

Figures 12 – 15 show the relative values in different states (n, e, ω) for the extended link model (MDP+) assuming self-similar call arrival process.

6.3 Results Analysis

From Table 2 the following conclusions are drawn:

- Although the self-similar traffic has the same mean inter-arrival time as the Poisson traffic, the reward loss for self-similar traffic is significantly higher since the burstiness of this arrival process is larger.
- The standard (MDP) and extended (MDP+) models yield similar reward loss since both CAC policies always accepts a new call as long as there is sufficient free capacity on the link (complete sharing).

From the graphs in Figures 6 and 7 the following conclusions are drawn:

- The relative value curve and the gain curve are identical for Poisson and self-similar call arrivals, due to the particular choice of λ, a and c parameters.
- The gain is maximal for $n = 0$ and drops as n increases.

From the graphs in Figures 8 – 11, which considers Poisson call arrivals, the following conclusions are drawn:

- Due to the memoryless property of the exponential inter-arrival and service distribution, the relative values are constant for different events e and offsets ω.

- The relative value $v(n, \pi)$ increases as the link state n increases.

From the graphs in Figures 12 – 15, which considers self-similar call arrivals, the following conclusions are drawn:

- The relative values $v(n, e, \omega, \pi)$ depends on the link state n, event type e and offset value ω, besides the CAC policy π.
- The relative value $v(n, e, \omega, \pi)$ is highest for $\omega = 0$ and drops as ω increases.
- An offset ω of zero is equivalent to a memoryless (Poisson) situation.
- Self-similar arrival processes give rise to offsets ω larger than zero, which yield smaller relative values (expected reward) and therefore larger reward loss, see Table 2.
- The relative value $v(n, e, \omega, \pi)$ for $e =$ *ARRIVAL* drops linearly as ω increases.
- The relative value $v(n, e, \omega, \pi)$ for $e =$ *DEPARTURE* drops faster than linearly as ω increases.

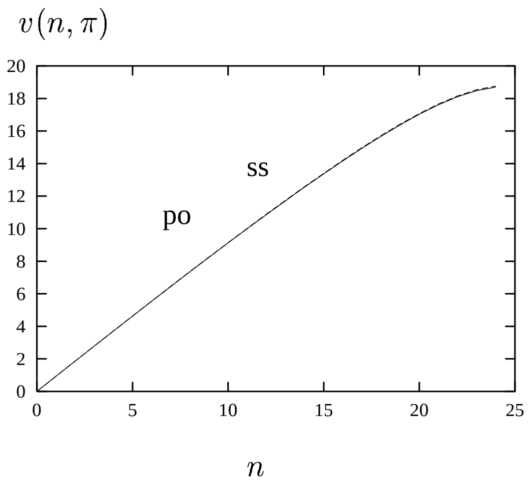

Figure 6: Relative value for standard MDP model for Poisson and self-similar call arrivals, respectively.

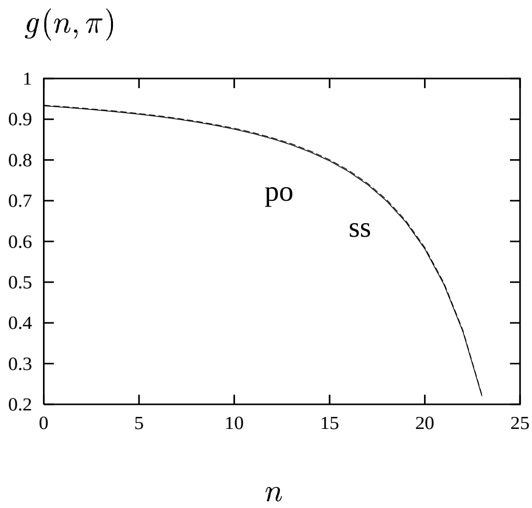

Figure 7: Link net-gain for standard MDP model for Poisson and self-similar call arrivals, respectively.

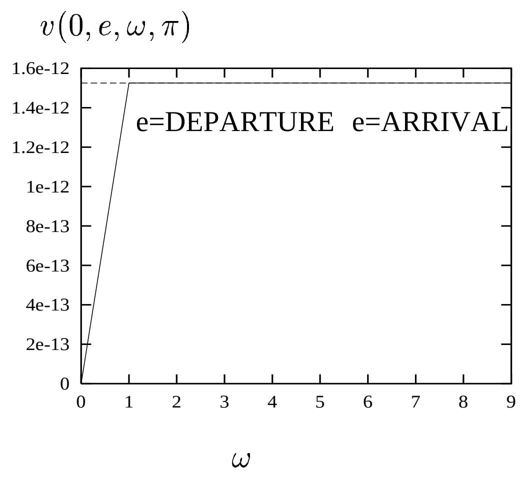

Figure 8: Relative values in $n = 0$ for MDP+ model with Poisson traffic.

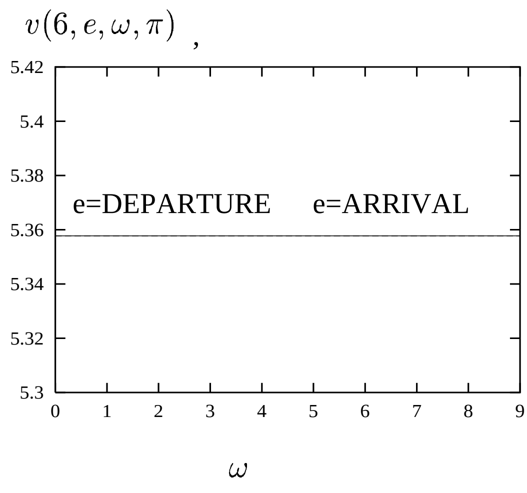

Figure 9: Relative values in $n = 6$ for MDP+ model with Poisson traffic.

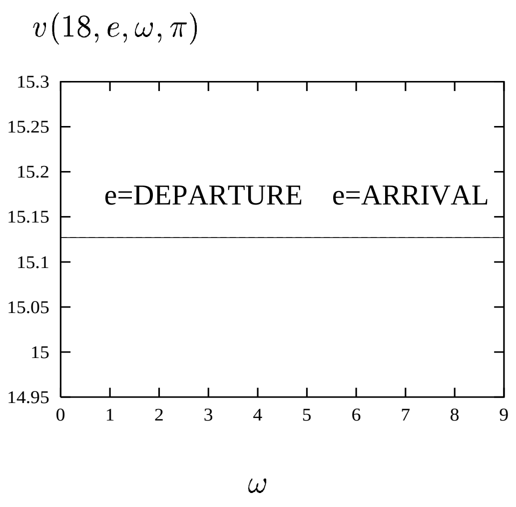

Figure 10: Relative values in $n = 18$ for MDP+ model with Poisson traffic.

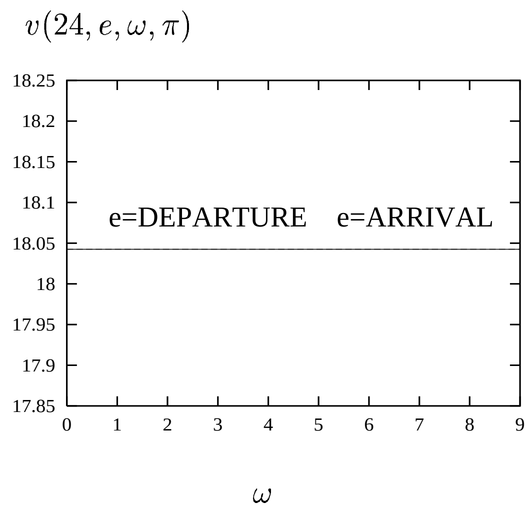

Figure 11: Relative values in $n = 24$ for MDP+ model with Poisson traffic.

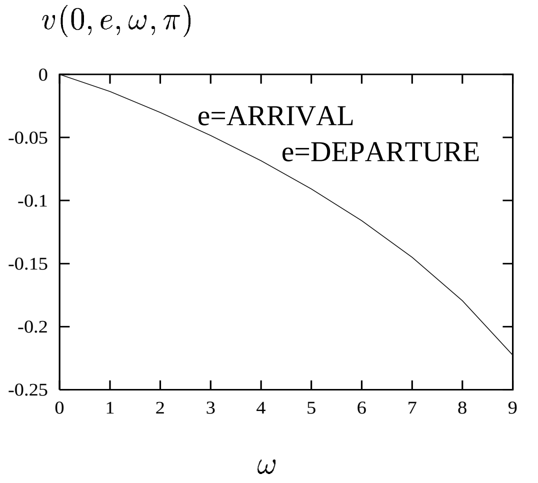

Figure 12: Relative values in $n = 0$ for MDP+ model with self-similar traffic.

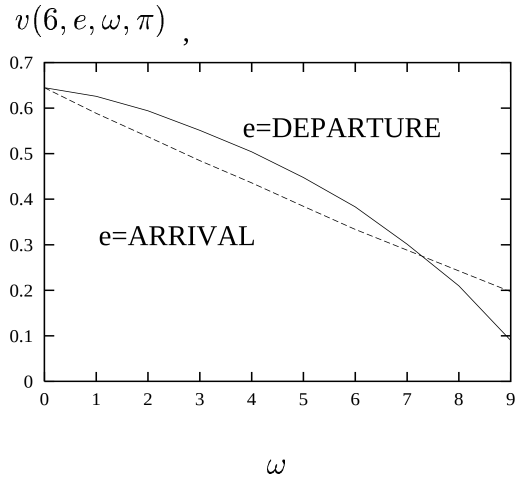

Figure 13: Relative values in $n = 6$ for MDP+ model with self-similar traffic.

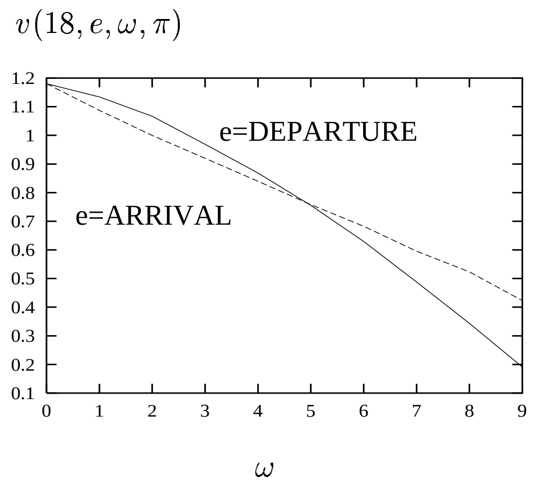

Figure 14: Relative values in $n = 18$ for MDP+ model with self-similar traffic.

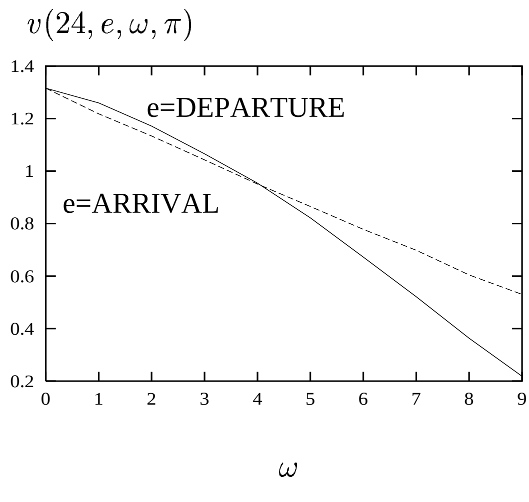

Figure 15: Relative values in $n = 24$ for MDP+ model with self-similar traffic.

7 DISCUSSION

In this section we discuss how to deal with multiple call categories. We also discuss how to model MDP-based CAC and routing on the network level, assuming self-similar call arrival processes to the origin-destination (OD) node pairs.

As we mentioned in the introduction, the superposition of two or more renewal processes is not, in general, a renewal process. Similarly to the $G = 1$ case, an exact SMDP formulation makes it necessary to keep track of the order of the G latest call events with unique category index as well as the time distance (offsets) between the events. For G categories, the exact state space will contain $S = (2G)!N_\omega^{2G-1}N$ states, where N denotes the number of simple states, $\mathbf{n} = (n_1, \cdots, n_G)$, on the link. The size of the state space, and the computational complexity, therefore increases exponentially with the number of categories. Already for $G = 2$, $N_\omega = 10$ and $N = 25$ we have $S = 600000$ states. The practical usage of the extended link MDP model is limited to the $G = 1$ case, or to the $G = 2$ case with small values of N_ω. There is no need to consult the extended link model to determine the optimal policy for $G = 1$. When $G = 1$ the optimal policy is to accept the call provided there is sufficient free capacity on the link (complete sharing).

On the network level, we have a set of OD node pairs which are offered a set of call arrival processes with unique bandwidth requirements and holding time distributions. Between each OD pair there is a set of alternative paths. The task of CAC and routing is to first determine the set of feasible paths between the OD pair which offers sufficient QoS to the new and existing calls. Second, the network should chose to reject the call or accept it on some path among the set of feasible paths. This choice should be consistent with GoS constraints as well as maximizing the average revenue rate for the operator.

Each given network link is offered traffic from a sub set, perhaps all, of the OD pairs. Assume that we have only one category with unique bandwidth requirement and exponential holding time distribution. Further, assume that the call arrival process to each OD pair sharing the given link is a renewal with Weibull distributed inter-arrival times. To obtain a link MDP model with feasible computational complexity we need to model the superposed call arrival process to the link. The Palm-Khintchine theorem states that the superposition of many independent and properly normalized renewal processes forms a Poisson process [15]. Hence, an exact or near-exact representation of the superposed arrival process, similar to the extended link MDP model for G categories, is only of interest for a limited range of network sizes, where relatively few OD pairs offer traffic to the link. Note that although the $G = 1$ case has a simple a priori solution on the link-level, this is not the case on the network level.

8 CONCLUSION

In this paper we formulated the CAC problem for a single link operating in loss mode. In this formulation each call category is characterized by its reward parameter defining the expected reward for carrying a call from this category. Such a formulation allows to apply Markov Decision Process (MDP) theory to solve the problem.

Traditionally, the MDP approach to CAC and routing has assumed Poisson call arrivals and exponentially distributed call holding times. These assumptions are reasonable for telephone calls. However, they become inaccurate for the TCP connections invoked within the World Wide Web (WWW) Internet service. In particular, measurements on real Internet traffic have revealed that the TCP connection arrival process is self-similar and that TCP connection holding time distribution is more heavy tailed than the standard exponential distribution.

This paper proposes an extended MDP model for self-similar call arrivals and exponential holding time distributions. We limit ourselves to exponential holding time distributions since this case is easier to handle than the case with heavy-tailed holding time distributions. Moreover, we consider only the one call category case. However, the model can easily be generalized to cope with multiple categories, albeit with large computational complexity. Numerical results show that the reward increase due to admission of a call (denoted link net-gain) depends on the time offset between the latest arrival and the latest departure.

Future work includes modelling of MDP-based CAC and routing on the network level, assuming self-similar call arrivals to the OD pairs.

References

[1] Ash G., *Dynamic Routing in Telecommunication Networks*, McGraw-Hill, 1998.

[2] Crovella M., Bestavros A., "Self-Similarity in World Wide Web Traffic – Evidence and Possible Causes", *IEEE/ACM Transactions on Networking*, Vol. 5, No. 6, pp. 835-846, 1997.

[3] Dziong Z., *ATM Network Resource Management*. McGraw-Hill (ISBN 0-07-018546-8), 1997.

[4] Dziong Z., Liao K-Q., Mason L.G., "Flow Control Models for Multi-Service Networks with Delayed Call Set Up", In *Proceedings of IEEE INFOCOM'90,*, pp. 39-46, IEEE Computer Society Press, 1990.

[5] Dziong Z., Mason L., "Call Admission and Routing in Multi-Service Loss Networks", *IEEE Trans. on Commun.*, Vol. 42, No. 2, 1994.

[6] Feldmann A., "Impact of Non-Poisson Arrival Sequences for Call Admission Control Algorithms with and without Delay", In *Proceedings of IEEE GLOBECOM*, pp. 617-622, Nov. 1996.

[7] Feldmann A., "Characteristics of TCP connection arrivals", Tech. Rep., AT&T Labs Research, 1998.

[8] Feldmann A., Gilbert A., Willinger A., and Kurtz T., "The Changing Nature of Network Traffic: Scaling Phenomena", *Computer Communication Review*, April 1998.

[9] Nordström E., "Near-Optimal Link Allocation of Blockable Narrow-band and Queue-able Wide-Band Call Traffic in ATM Networks", In *Proceedings of ITC 15*, Washington, 1997.

[10] Nordström E., "Near-Optimal CAC and routing for multi-service networks by Markov decision theory", In *Proceedings of KK foundation conference*, Skövde, Sweden, April, 2002.

[11] Nordström E., "CAC and Routing of Self-Similar Call Traffic based on Markov Decision Theory", Visby, Sweden, May, 2003.

[12] Nordström E., Carlström J., “A New Reward Model for MDP State Aggregation with Application to CAC and Routing”, Accepted for publication in *Euorpean Transactions on Telecommunications*, 2004.

[13] Nordström E., Dziong Z., “CAC and Routing for Multi-Service Networks with Blocked Wide-band Calls Delayed, Part I: Exact MDP Framework”, submitted, 2004.

[14] Nordström E., Dziong Z., “CAC and Routing for Multi-Service Networks with Blocked Wide-band Calls Delayed, Part II: Approximate MDP Framework”, submitted, 2004.

[15] Willinger W., Paxson V., Reidi R., and Taqqu M., “Long-range Dependence and Data Network Traffic”, *Long-range Dependence: Theory and Applications*, Doukhan P., Oppenheim G., and Taqqu M., eds, Birkhauser, 2002.

[16] Tijms H., *Stochastic Modeling and Analysis – a Computational Approch*, Wiley, 1986.